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# Use of Graph Theory for Applications, Representations, Modeling and Problem Solving in Mathematics and other fields

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**Abstract**— Graph theory, along with the practical applications of graph labeling and graph coloring is a very important field of mathematics as well as mathematical modeling. The significance of graph theory is growing as research becomes more model oriented as well as illustration oriented. Graph theory is widely used to prove several mathematical theorems and models for proper understanding and further research. The paper further details various uses and techniques used of graph theory that are used to solve problems in varied fields of science and technology other than mathematics. These fields include genomics in biochemistry, genetics in biology, network communications and coding in electronics engineering, representation of algorithms in computer science as well as a wide variety of usage in scheduling, production planning, quality control and logistics in the field of supply chain management and operations research. Graph labeling and graph coloring are also very useful in the representation of financial and investment models, travel and tourism industry, explaining various management models and thus advanced complex business models. Since graph labeling and graph coloring are very important tools for representation of real time models and thus functions in any field of research, they can be used for creating novel problem solving techniques as well. The paper thus focuses on the diverse aspects of this powerful method of representation of scientific facts that can be used to solve many real time problems. The following paper introduces the reader with the history, introduction, terminology and definitions of graph theory. The second part of the paper deals with the introduction of applications of graph theory in the diverse fields of science and technology.

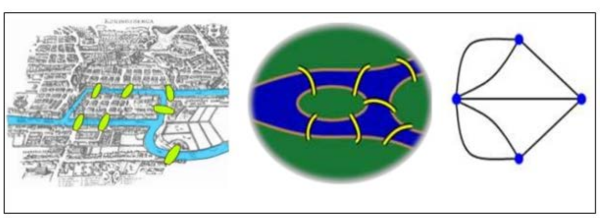
**Index Terms**— Coloring Applications, Graph Coloring, Graph Labeling, Modeling, Problem Solving Techniques, Representation.



## 1 Introduction

### History of Graph Theory:

Graph theory as a separate branch of mathematics started developing around 280 years ago in the town of Konigsberg, Germany by a mathematician called Leonhard Euler. Euler's representation of the Konigsberg Bridge problem is considered to be the first theorem of graph theory on the seven bridges of Konigsberg and was published in 1736.



Following that, in 1771, a French mathematician called Alexandre-Théophile Vandermonde wrote a paper on the Knight problem but also dealt with certain problematic aspects of Leonhard Euler's paper. Vandermonde's theory developed further into the Knot Theory and started the green shoots of a branch of mathematics which would be later known as topology. Also, Euler's formula relating to the number of edges, vertices and face of a convex polyhedron was studied and generalized by another French mathematician called Augustin Luois-Cauchy in 1811 which officially represents the beginning of the branch of Mathematics known as Topology.

In 1847, the German physicist called Gustav Kirchhoff developed the theory of trees for their applications in electrical networks. Ten years later, A British Mathematician called Arthur Cayley created the structure known as the Cayley tree which is a tree in which each non-leaf graph vertex has a constant number of branches $n$ is called an $n$-Cayley tree. 2-Cayley trees are path graphs. The unique $n$-Cayley tree on $n+1$ nodes is the star graph. This structure was derived when Cayley was studying some analytical forms of a class of graphs that are called trees. This study led to the development of the analysis of theoretical chemistry. Along with theoretical chemistry, the concept of enumeration of graphs was also established which was later used by the Hungarian Mathematician George Polya between 1935 to 1937.

It is also worth mentioning that about the time when Kirchoff and Cayle were establishing the principles of graph theory by their mathematical analysis and research in the field of electric circuits and the tree graphs, another important milestone in

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graph theory were laid. This was the four-color conjecture, which states that four colors are sufficient for coloring any Atlas such that countries with common boundaries have different colors.

Also, A.F. Mobius gave the idea of complete graph and bipartite graph and Kurdatowwski proved that they are planar by means of recreational problems. In 1913, H. Dudeney mentioned a puzzle problem. Even though the four color problem was invented, it was solved only after a century by Kenneth Apple and Woltgang Haken.

Later on in 1959, a Dutch Mathematician Nicolaas Govert de Bruijn inked the results on trees with the chemical composition of materials which was an advandement og analytical chemistry. The combination of graph theory to study the composition of materials is considered to be a great step in the development of the analysis of materials.
The first dedicated textbook on graph theory was published by Denes Konig is the year 1936. After that, Frank Harary published a book which is considered as a connecting tool for various academic communities such as mathematicians, engineers, chemists and physicists.

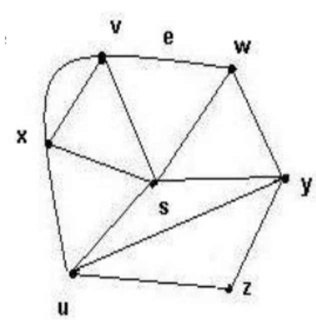
## 2 Definition of Terms in Graph Theory

### 2.1 Graph

A graph is a mathematical abstraction that is useful for solving many kinds of problems. Fundamentally, a graph consists of a set of vertices, and a set of edges, where an edge is something that connects two vertices in the graph. A graph is a pair (V,E), where V is a finite set and E is a binary relation on V. V is called a vertex set whose elements are called vertices. E is a collection of edges, where an edge is a pair (u,v) with u,v in V.

### 2.2 Graph G

(V, E) is a collection of V nodes connected by E links. This definition of a graph is vague in certain respects; it does not say what a vertex or edge represents. They could be cities with connecting roads, or web-pages with hyperlinks.




### 2.3 Path

A path is a simple graph whose vertices can be ordered so that two vertices are adjacent if and only if they are consecutive in the list.

### 2.4 Undirected Graph

A graph in which each edge symbolizes an unordered, transitive relationship between two nodes is called an undirected graph. Such edges are rendered as plain lines or arcs.

### 2.5 Directed Graph/ Diagraph

A graph in which each edge symbolizes an ordered, non-transitive relationship between two nodes. Such edges are rendered with an arrowhead at one end of a line or arc.

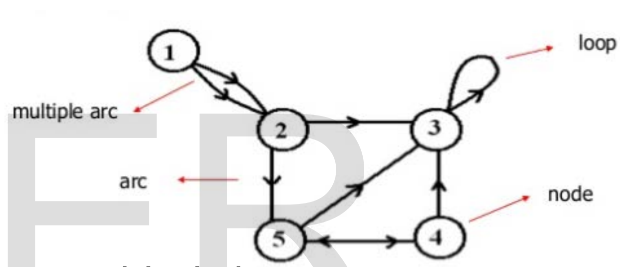
### 2.6 Loop

A loop is a special type of edge that connects a vertex to itself. Loops are not used much in street network graphs.

### 2.7 Degree

Degree is the number of edges which connect a node.

a. In Degree: Number of edges pointing to a node.
b. Out Degree: Number of edges going out of a node.

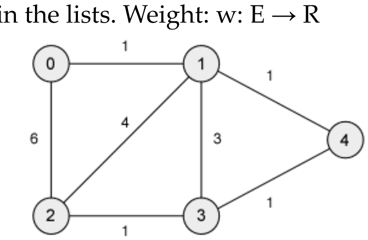



### 2.8 Un-weighted Edge

A graph in which all the relationships symbolized by edges are considered equivalent is called an un-weighted edge. Such edges are rendered as plain lines or arcs.

### 2.9 Weighted Edge

Weighted edges symbolize relationships between nodes which are considered to have some value, for instance, distance or lag time. Such edges are usually annotated by a number or letter placed beside the edge. If edges have weights, we can put the weights in the lists. Weight: w: E → R




### 2.10 Tree

An undirected connected graph T is called tree if there are no cycles in it. There is exactly one simple path between any vertices u and v. Simple path: Simple path is a path in which all the vertices are distinct.

### 2.11 Spanning Tree

A sub graph T of a connected graph G, which contains all the vertices of G and T is called a spanning tree of graph G. It is called spanning tree because it spans over all vertices of graph G.

## 3 Applications of Graph Theory

The graphical representations and the models which are produced by the theorems of graph theory are used in diverse fields of science and technology. The major areas that use graph theory extensively for problem solving and solution design are Infrastructure networks, Business model representations, Chemistry, Physics, Statistical Physics, Computer network security, Map coloring, GSM Mobile network, Wireless networks, Biology and Genetics.

But before going into the details of such applications, it is noteworthy to understand the Shortest Path Algorithm which is employed in a variety of solution building models. The Shortest Path Algorithm is the solution which solves the problem of finding a path between two vertices or nodes in a graph so that the sum of the weights of its constituent edges is minimized.

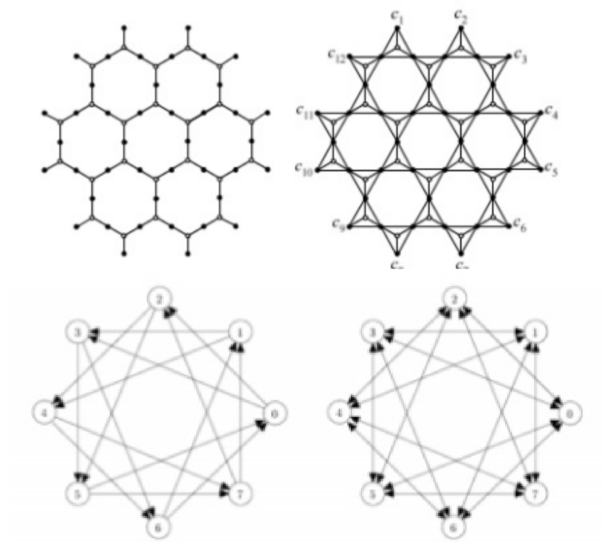
Most of the practical networks in science and technology such as road networks, railway networks, logistical networks and distribution systems face this typical problem of finding the shortest path between two vertices. The shortest path algorithm along with its various mathematical variations is used to solve such problems. These variations are as follows:

- The single-source shortest path problem, where the task is to find shortest paths from a source vertex $v$ to all other vertices in the graph.
- The single-destination shortest path problem, where the task is to find shortest paths from all vertices in the directed graph to a single destination vertex $v$. This can be reduced to the single-source shortest path problem by reversing the arcs in the directed graph.
- The all-pairs shortest path problem, where the task is to find shortest paths between every pair of vertices $v$, $v'$ in the graph.

These generalizations and sometimes the combinations of more than one type of variations can lead to the creation of significantly more efficient algorithms than the simpler approach of running a single-pair shortest path algorithm on all the relevant nodes in consideration.

### 3.1 Wireless Networks

Graph theory is finding an increasingly effective usage in wireless multi-hop networks in various forms such as wireless sensors networks, underwater sensor networks, vehicular networks, mesh networks and UAV (Unarmed Aerial Vehicles) formations.



It is also used in ad-hoc networks, hybrid networks, delay tolerant networks and intermittently connected networks for applications in military and civilian applications.

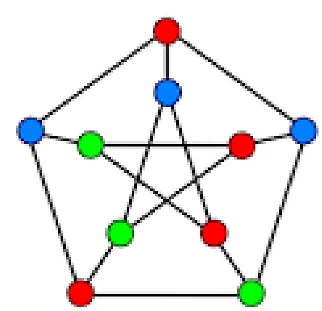
### 3.2 Computer Applications

Graph theory is extensively used in computer applications in the development of computer algorithms. Numerous algorithms are used to solve problems that are modeled in the form of graphs. Some particularly useful algorithms are as follows:

a. Shortest path algorithm in a network,
b. Finding a minimum spanning tree,
c. Finding graph planarity,
d. Algorithms to find adjacency matrices,
e. Algorithms to find the connectedness,
f. Algorithms to find cycles in graph,
g. Algorithms for searching an element in a data structure.

### 3.3 Vertex Coloring

Vertex Coloring is one of the most important concepts in graph theory and is used in many real time applications in computer science. Various coloring methods are available and can be used on a requirement basis. The proper coloring of a graph is the coloring of the vertices and edges with minimal number of colors such that no two vertices have the same color. The minimum number of colors is called the Chromatic Number and the graph is called a properly colored graph.



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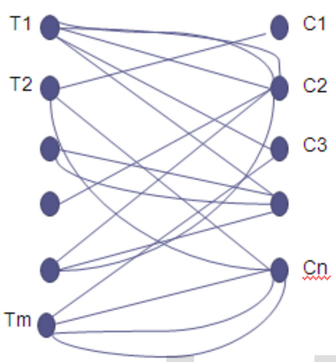
### 3.3 Time Table Scheduling

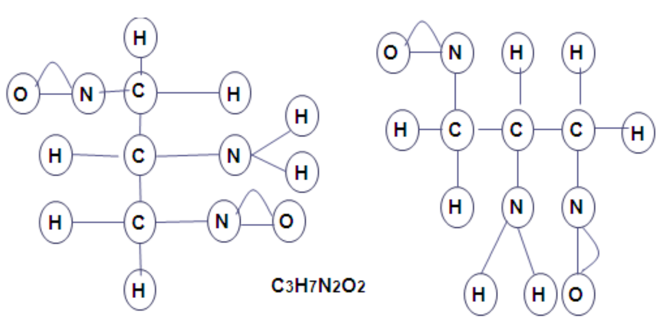
Graph theory also plays an important role in finding the best possible schedule for time tables for transportation networks, shipping networks, port dispatch networks, city bus routes, train routes, air-traffic control and various other supply chain and logistics networks. Graph theory is used to represent various business models and concepts which are extremely useful. Any graphical representation of a business problem helps derive a long-lasting and effective solution.






### 3.4 Graph Theory in Chemistry

Graphs are used in the field of chemistry to model chemical compounds and their structures. In computational biochemistry, same sequences of cell samples have to be excluded to resolve the conflicts between two sequences. This is modeled in the form of graph where the vertices represent the sequences in the sample. An edge will be drawn between two vertices if and only if there is a conflict between the corresponding sequences. The aim is to remove possible vertices to eliminate all conflicts.




### 3.5 Graph Theory in Physics

In physics, graphs are used in condensed matter physics. It is usual to describe a solid state and molecular systems as tight binding models. Graph theory is also widely used in the field of statistical physics. Statistical Physics in a branch of science which deals with the usage methods of probability theory and statistics, and particularly the mathematical tools for dealing with large populations and approximations, in solving physical problems. The major areas of statistical physics that employ graph theory are statistical mechanics, particle physics and statistical analysis of thermodynamic problems and results.

### 3.6 Graph Theory in in Biology and Genetics

Graph theory is increasingly used as a very effective problem solving technique in the ever-evolving field of biology, genomics, biotechnology, bioinformatics, biological networks and genetics. The representation of complex DNA structures and the derivation of the functions of various genes is a very effective tool for molecular biologists. Advanced graphical models are also used widely now for the purpose of molecular biology, computational biology and protein analysis. The most important concepts of graph theory in the field of biology and genetics are Fragment assembly, Overlap graphs, sequence comparison, Alignment-free method and Weighted directed graph.

## 4 Conclusion

Extensive as well as intensive usage of graph theory including the techniques of graph labeling and graph coloring to represent and furthermore solve problems by creating proper problem solving techniques are a very important tool in many major fields of basic sciences and modern technology. Spanning tree representations and solution based representations by using the shortest path problem, the single-source shortest path problem, the single-destination shortest path problem, the all-pairs shortest path problem and other generalizations of graphical representations are very effective methods to represent and solve problems of extremely diverse fields. These fields include but are not limited to various network management areas such as transportation networks, logistic networks, operations networks, wireless networks and biological networks. Graph labelling is also a very effective tool in solving problems in genomics, physics, chemistry, biology, genetics and computer and telecommunication applications.

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